IV Semester B.A./B.Sc. Examination, May/June 2018
(CBCS) (Fresh + Repeaters) (2015 – 16 and Onwards)
(Semester Scheme)
MATHEMATICS (Paper – IV)

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** Parts.*

PART – A

1. Answer **any five** questions. **(5×2=10)**
   a) Define a normal subgroup.
   b) If $f : G \to G'$ is a homomorphism then prove that $f(e) = e'$, where e and e′ are the identity elements of G and G′ respectively.
   c) Expand $f(x) = x$ in half range cosine series over the interval $(0, \pi)$.
   d) Show that $f(x, y) = x^3 + y^3 - 3xy + 1$ is minimum at (1, 1).
   e) If $L[f(t)] = F(s)$, then show that $L[e^{at} f(t)] = F(s - a)$.
   f) Find $L[t \sin t]$.
   g) Solve : $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 0$.
   h) Prove that 'x' is a part of the complementary function of $x^2 \dfrac{d^2y}{dx^2} - 2x(x+1)\dfrac{dy}{dx} + 2(x+1)y = x^3$.

PART – B

Answer **one full** question. **(1×15=15)**

2. a) Prove that a subgroup H of a group G is normal subgroup of G iff $gHg^{-1} = H, \forall g \in G$.
   b) Define centre of a group and prove that the centre of a group G is a normal subgroup of G.
   c) If $f : G \to G'$ be a homorphism from the group G into G′ with Kernal K, then prove that f is one-one iff $K = \{e\}$, where 'e' is the identity element in G.

OR

P.T.O.

3. a) Prove that the intersection of two normal subgroups of a group is a normal subgroup.

   b) If G is a group and H is a subgroup of index 2 in G, then show that H is a normal subgroup of G.

   c) State and prove Fundamental theorem of Homomorphism.

PART – C

Answer **two full** questions. **(2×15=30)**

4. a) Find the Fourier expansion of $f(x) = x - x^2$ in $(-1, 1)$.

   b) Obtain half range sine series of $f(x) = \sin x$, $0 < x < \pi$.

   c) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ by Taylor series upto 3rd degree terms.

OR

5. a) Find the Fourier series of $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ \pi - x, & \pi \le x \le 2\pi \end{cases}$.

   b) Find the extreme values of the function $f(x) = x^3y^2(1 - x - y)$.

   c) Show that minimum value of $x^2 + y^2 + z^2$ subjected to the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ is 27.

6. a) Find $L[e^t \sin^2 t]$ and $L[\sinh^2 at]$.

   b) Find $L[f(t)]$ if $f(t) = \begin{cases} 2t, & 0 \le t \le 5 \\ 1, & t > 5 \end{cases}$.

   c) Using Convolution theorem find $L^{-1}\left[\frac{1}{(s+2)(s+4)}\right]$.

OR

7. a) Find :

   i) $L[t^3e^{-3t}]$.

   ii) $L[e^{-t}(2\cos 5t - 3\sin 5t)]$.

b) Find $L^{-1}\left[\frac{s^2}{(s-1)(s^2+1)}\right]$.

c) Find $L[t^2 u(t-3)]$ using convolution property.

PART – D

Answer **one full** question. **(1×15=15)**

8. a) Solve : $(D^2 - 5D + 6)y = e^{4x} + \sin 2x$.

b) Solve : $4x^2 y'' + 4xy' - y = 4x^2$.

c) *Solve : $xy'' - (1 + x)y' + y = 0$, given that $(x + 1)$ is a part of complementary* function.

OR

9. a) Solve : $\frac{d^2y}{dx^2} + y = e^{-x} + 5x^2e^x$.

b) Solve : $\frac{dx}{dt} = 3x - y$; $\frac{dy}{dt} = x + y$.

c) Solve : $y'' + y = \tan x$ by the method of variation of parameters.

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